Chapter 4: Statistical Foundations

## Demonstrate knowledge of the characteristics of return distributions.

1. Recognize ex ante and ex post return distributions
2. Recognize the importance of the normal distribution in statistical analysis
3. Describe the characteristics of lognormal distributions

## Demonstrate knowledge of moments of return distributions (i.e., mean, variance, skewness, and kurtosis).

1. Explain the first four raw moments of return distributions
2. Explain the central moments of return distributions
3. Explain skewness of return distributions
4. Explain kurtosis and excess kurtosis of return distributions
5. Describe the characteristics of platykurtic, mesokurtic, and leptokurtic distributions

Mesokurtic: same distribution with normal distribution

Leptokurti: positive excess kurtosis

Platykurtic: negative excess kurtosis

## Demonstrate knowledge of various measures of correlation of returns.

1. Recognize the importance of correlation in alternative investment portfolio management

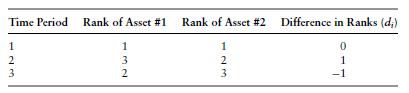
Correlation affects diversification, and the diversification drives the risk of a portfolio of assets relative to the risks of the portfolio’s constituent assets

1. Define and calculate covariance
2. Define and calculate correlation coefficient

It measures the degree of association between two variables

1. Define and calculate the Spearman rank correlation coefficient

The spearman rank correlation is a correlation designed to adjust for outliers by measuring the relationship between variable ranks rather than variable values.





It is preferred because of the way it handles the effects of outliers (extremely high or low data values)

1. Discuss the role of correlation in portfolio diversification

The reduction in risk is found by combining assets that are not perfectly positively correlated. The greatest risk reduction occurs when the assets’ correlation coefficient is -1. Ultimate diversification comes with no volatility

1. Define and calculate beta in the context of the CAPM

Beta of an asset is defined as the covariance between the asset’s returns and a return such as the market index, divided by the variance of the index’s return

Beta indicates the responsiveness of asset I to fluctuations in the values of the market portfolio, or responsiveness of asset I to fluctuations in the benchmark.

1. Define and calculate autocorrelation

Autocorrelation of a time series of returns from an investment refers to the possible correlation of returns with one another through time. For example, positive first-order autocorrelation is above average, return in t-1 tends to be followed by an above-average return in time t

First-order autocorrelation coefficient =

Note: autocorrelation would be zero in a perfectly efficient market

**Good**: how do we know that log returns will be roughly normally distributed over reasonably long periods of time if the returns have no autocorrelation and if very return on any asset over a long time period such as a month is the sum of the log returns of the sub-periods. Even if the returns over extremely small units of time are not normally distributed, the central limit theorem indicates that the returns formed over longer periods of time by summing the independent returns of the sub-periods will tend toward being normally distributed.

1. Define and apply the Durbin-Watson test

Searching for the presence of first-order autocorrelation in time series

DW value of 2 means no significant autocorrelation

DW >>2 (e.g. DW>3): negative autocorrelation

DW <<2 (e.g. DW<1): positive autocorrelation

## Demonstrate knowledge of standard deviation (volatility) and variance.

1. Define and explain return standard deviation (volatility)

Standard deviation is the typical amount by which an investment’s actual return deviates from its average

1. Describe the properties of return variance and standard deviation and do calculation

The variance of return of a portfolio p

For perfectly correlated assets, the standard deviation of the portfolio p is

The variance of weekly return is the sum of variances of daily returns

when

The standard deviation of the levered position can be approximated using unlevered asset

For example, , then

For multiple period standard deviation,

when

when

## Demonstrate knowledge of methods used to test for normality of distributions.

1. Recognize the three main reasons for non-normality observed in alternative investment returns (i.e., autocorrelation, illiquidity, and nonlinearity), and discuss the effect of each on returns

There are 3 main reasons for non-normality in alternative investment returns: autocorrelation, illiquidity, and nonlinearity. The impact of each on returns is listed below:

**Autocorrelation**: highly dispersed longer-term returns

**Illiquidity**: In illiquid markets, prices are often estimated by models and professional judgement rather than by competitive market prices. As a result, returns are smoothed and tend to exhibit less volatility

**Non**-**linearity**: highly non-symmetric return distribution over long time intervals.

1. Discuss tests for normality that use sample moments

By testing skewness and kurtosis

1. Recognize and apply the Jarque-Bera test

JB test involves a statistic that is a function of skewness and excess kurtosis

JG

where is the # observations, is the skewness of the sample, and is the excess kurtosis

the null hypothesis is: underlying distribution is **normal** and JB is equal to **zero**. Thus the JB test for normality is whether the test statistic is large enough to reject the null hypothesis of normality. The JB test is more powerful when the #observation is large.

For example, JB=2.219. statistical confidence of 95%. The critical value is 5.99 >2.219, as such we cannot reject the null hypothesis of normality.

## Demonstrate knowledge of time-series return volatility models.

1. Identify various measures used in time-series models (e.g., price levels, price variation, risk)

GARCH: generalized autoregressive conditional heteroskedasticity

1. Define the concepts of heteroskedasticity and homoscedasticity

Heteroskedasticity: variance of variable changes with respect to a variable, such as time

Homoscedasticity: variance of a variable is constant.

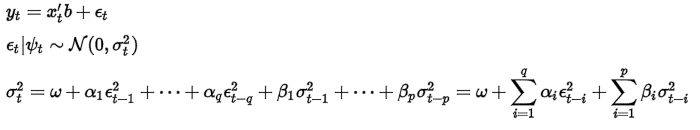
1. Recognize the key components of the generalized autoregressive conditional heteroskedasticity (GARCH) method

GARCH method allows for heteroskedasticity and can be used when it is believed that risk is changing over time.

Unconditionally heteroskedastic: a financial asset exhibits a clear pattern of return variation

Conditionally heteroskedastic: different levels of return variation

1. Describe how the GARCH method is used to model risk evolution through time



1. Contrast the GARCH method with the autoregressive conditional heteroskedasticity (ARCH) method

ARCH allows future variances to rely only on past disturbances, whereas GARCH allows future variances to depend on past variances as well.

